Solve Continuous Time Hotelling Extractive Industry.

J. Hartwick
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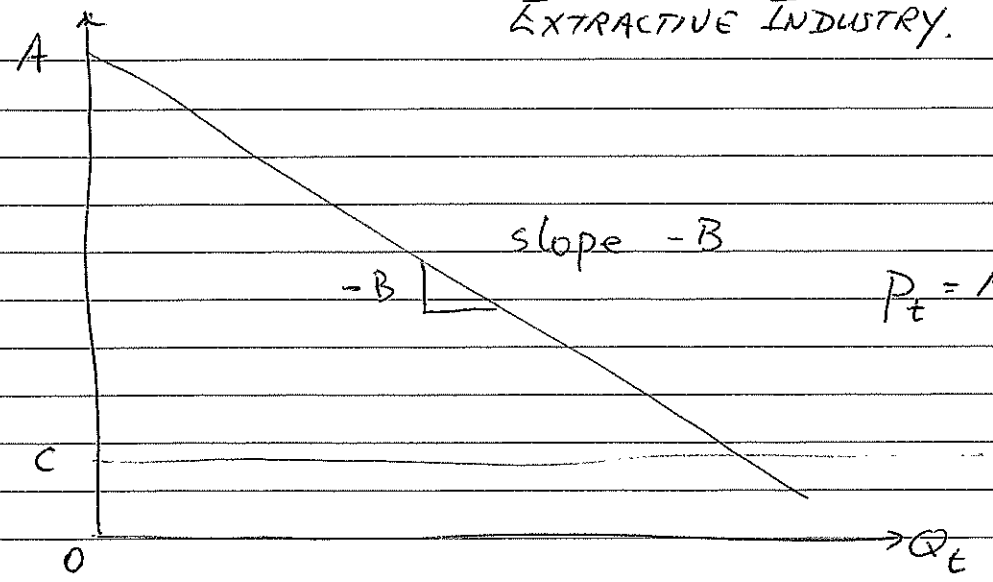

Solve working r% backwards on rent from $Q_T = 0$

$$[A-c]e^{-r[T-t]} = P_t - c$$

$$= A - BQ_t - c.$$

Thus

$$Q_t = \frac{A-c}{B}\left\{1 - e^{-rT}\, e^{rt}\right\}$$

$$S_0 = \int_0^T Q_t\, dt = \int_0^T \frac{A-c}{B}\, dt - \frac{A-c}{B} e^{-rT} \int_0^T e^{rt}\, dt$$

$$= \left[\frac{A-c}{B}\right]T - \frac{A-c}{B} e^{-rT} \frac{1}{r}\left[e^{rT} - 1\right]$$

$$S_0 = \frac{A-c}{B}\left\{T - \frac{1}{r} + \frac{e^{-rT}}{r}\right\}$$

Set $A = 100$, $B = \frac{1}{2}$, $c = 10$, $r = 0.1$, $S_0 = 400$

Solve with MAPLE program.

$T = 7.49726$

$Q_0 = 94.95076$

See attached MAPLE program

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```
> restart; S0:=400;r:=0.1;c:=10;B:=0.5;A:=100;
```

$$S0 := 400$$

$$r := 0.1$$

$$c := 10$$

$$B := 0.5$$

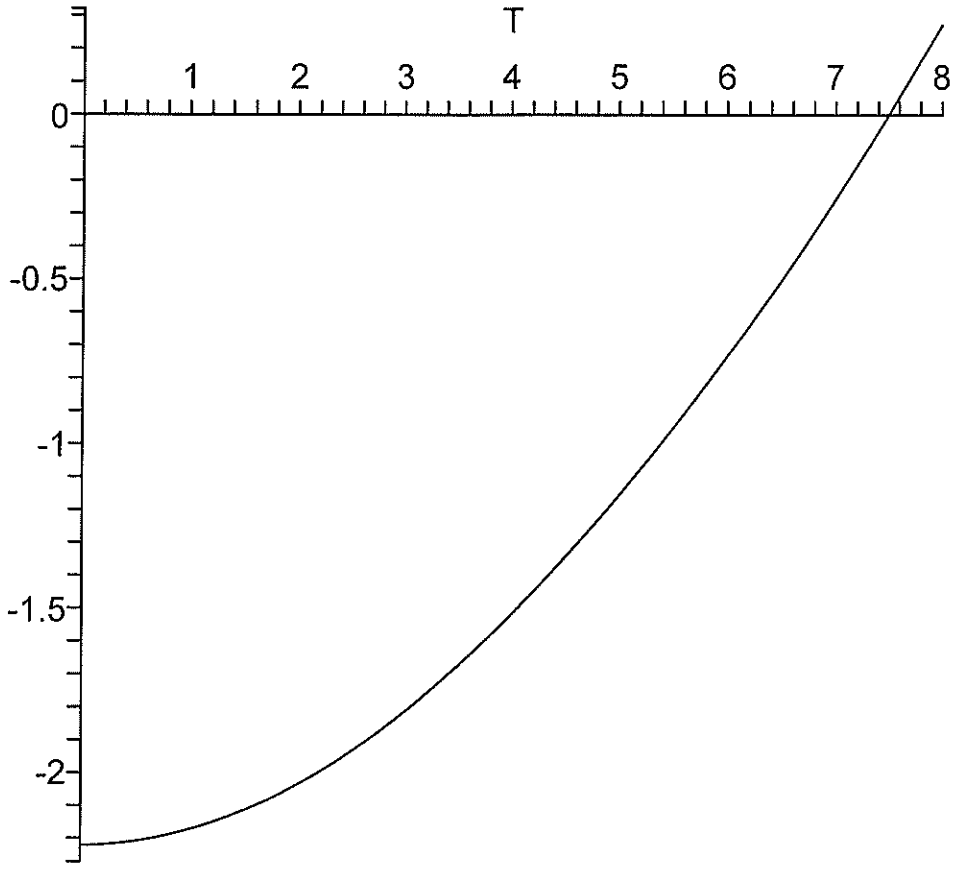
$$A := 100$$

MAPLE SOLVING for T for Hotelling Linear Demand

```
> f:=T->(T-(1/r)+(1/r)*exp(-r*T))-(S0*B)/(A-c);
```

$$f := T \to T - \frac{1}{r} + \frac{e^{(-r\,T)}}{r} - \frac{S0\,B}{A - c}$$

```
> plot(f(T),T=0.01..8);
```




```
> solve((T-(1/r)+(1/r)*exp(-r*T))-(S0*B)/(A-c)=0,T);
```

7.497264229, -6.001257813

```
> Q0:=((A-c)/B)*(1-exp(-r*7.497264229));
```

$$Q0 := 94.95075616$$

```
> # check: takes about 7.5 time periods with say about 95/2 units on averag
  per period: 356.25 tons (approx 400 tons)
```

check result.